The ``Synchronous´´ Painter

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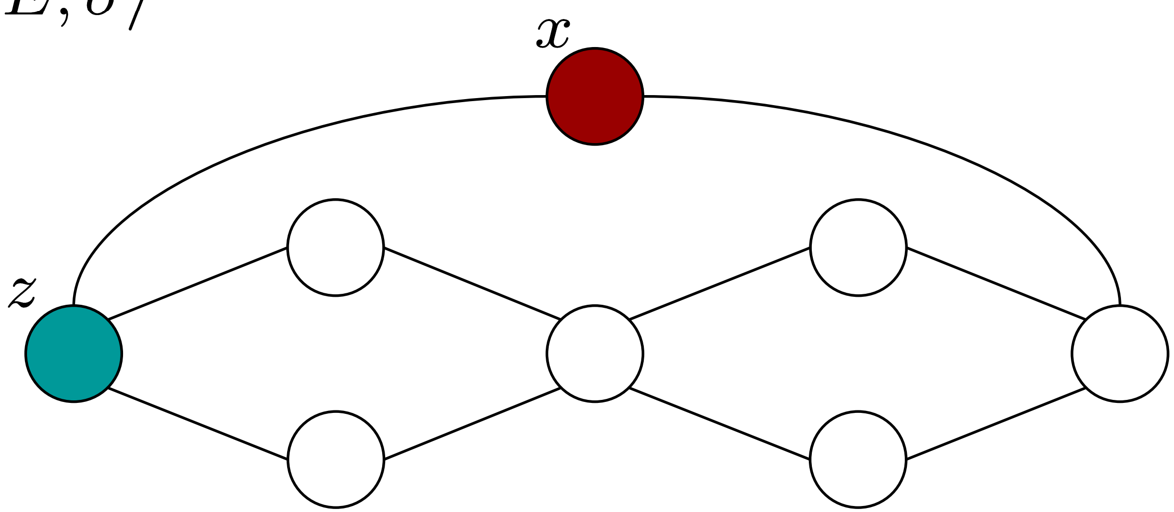
Idea

Rephrase causality analysis as a Graph Colouring Problem.

Explain the step responses of Statecharts in terms of colourings.

Graph Colouring

$$M = \langle V, E, \sigma \rangle$$



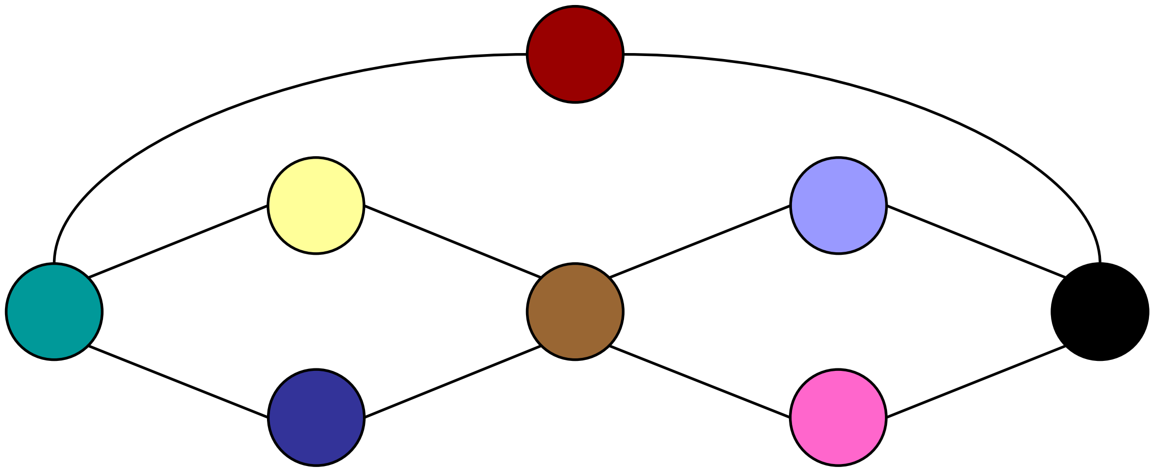
$$K = \{Green, Red, Blue, \ldots\}$$

$$\sigma(z) = \{Green\}$$

$$\sigma(x) = \{Red\}$$

Graph Colouring

$M = \langle V, E, \sigma \rangle$

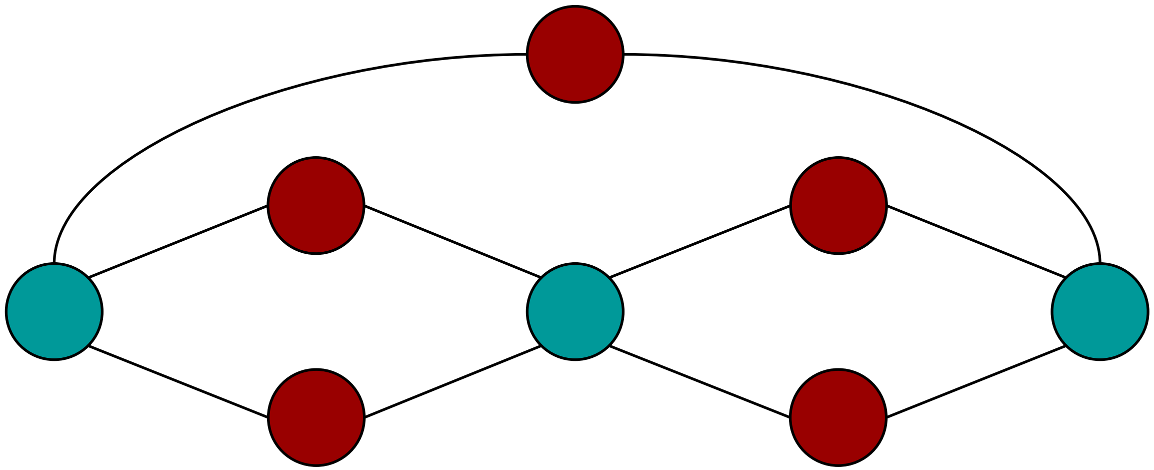


$K = \{Green, Red, Blue, \ldots\}$

$Green \vee Red \vee \ldots \models k_i \Rightarrow \Box(\neg k_i)$

Graph Colouring

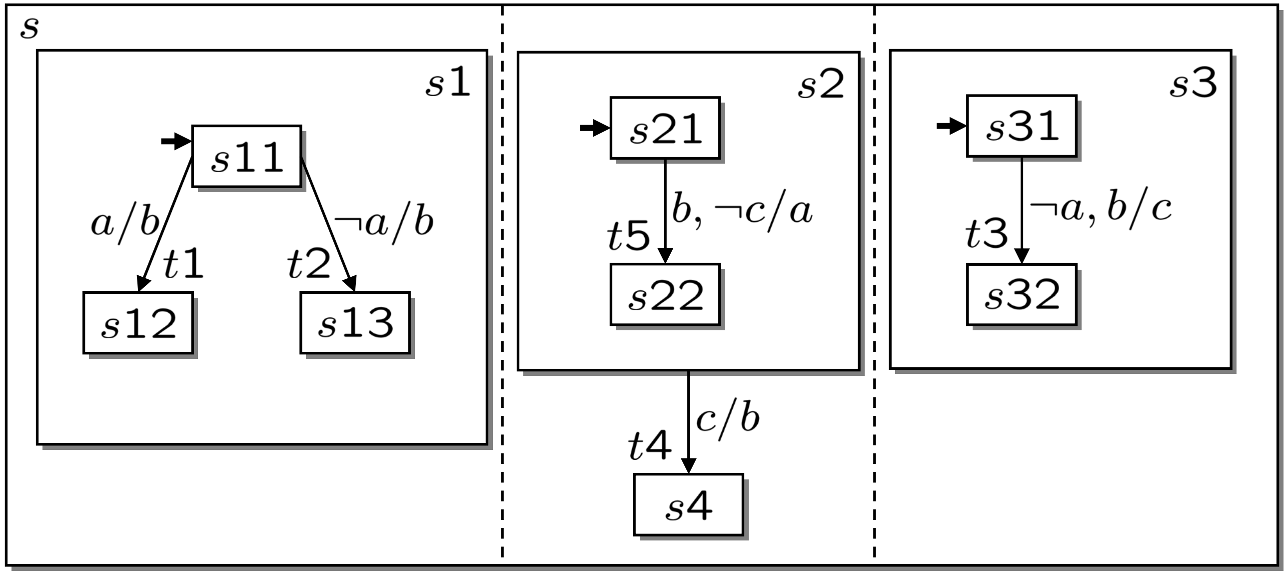
$$M = \langle V, E, \sigma \rangle$$



$$Green \vee Red \models Green \Rightarrow \Box(Red)$$

$$Green \vee Red \models Red \Rightarrow \Box(Green)$$

A-mazing Components



$\Phi :=$

$t1 \supset b \wedge t2 \supset b \wedge t3 \supset c \wedge t4 \supset b \wedge t5 \supset a \wedge$

$(s11 \wedge a \wedge \neg t2) \supset t1 \wedge$

$(s11 \wedge \neg a \wedge \neg t1) \supset t2 \wedge$

$(s31 \wedge \neg a \wedge b) \supset t3 \wedge$

$(s2 \wedge c) \supset t4 \wedge$

$(s21 \wedge b \wedge \neg c \wedge t4) \supset t5$

- flat conjunction of transitions
- obtained from visual syntax, structurally and incrementally
- negations code non-determinism, priorities and hierarchy

A-mazing Components

$$\Phi := t_1 \wedge t_2 \wedge t_3$$

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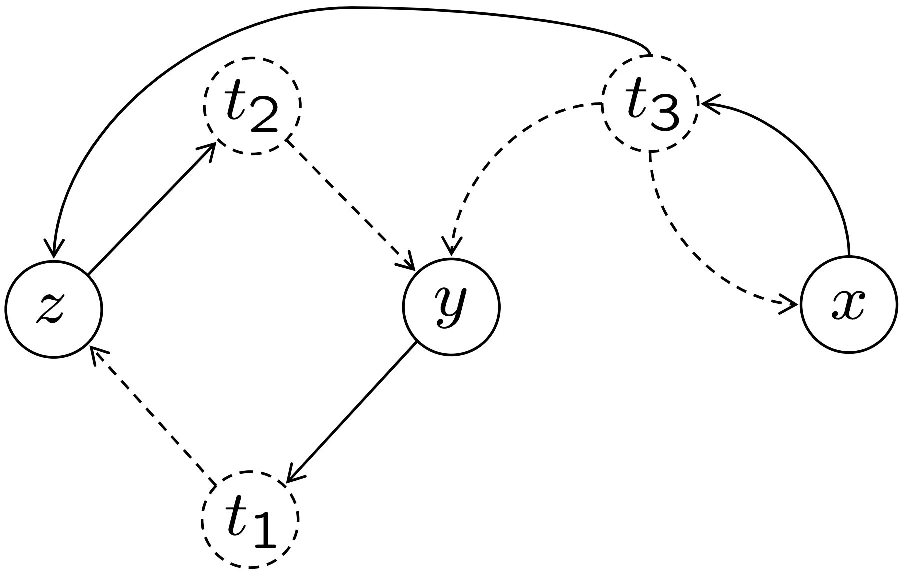


Words in Colour

Let $K = (k_i)_{i \in \mathbb{N}}$ be a family of colours.

The construction of $Form(K)$ is done by means of $\wedge, \vee, \supset$ and$\neg, \Box^\gamma$ and $\Diamond^\gamma$.

The interpretation of the extensional connectives is done in the familiar (truth-table) way with respect to a world (in a model).

$$M, w \models \Box^\gamma \varphi \quad \text{iff} \quad \forall (w, w') \in E^\gamma.\ M, w' \models \varphi$$
$$M, w \models \Diamond^\gamma \varphi \quad \text{iff} \quad \exists (w, w') \in E^\gamma.\ M, w' \models \varphi$$

Colour Patterns

Semantic consequences of the form:
$\Gamma \models \Delta \Rightarrow \varphi$ are called a *colour-patterns*

$\Gamma \models \Delta \Rightarrow \varphi$

$$(\forall \psi \in \Gamma.\ \langle V, E, \sigma \rangle \models \psi) \Rightarrow$$
$$\forall w.\ (\forall \delta \in \Delta.\ \langle V, E, \sigma \rangle, w \models \delta) \Rightarrow \langle V, E, \sigma \rangle, w \models \varphi$$

which indeed specifies a class consisting of all
$\langle V, E, \sigma \rangle, w \models \varphi$ that fulfil the *axioms* of Γ, Δ.

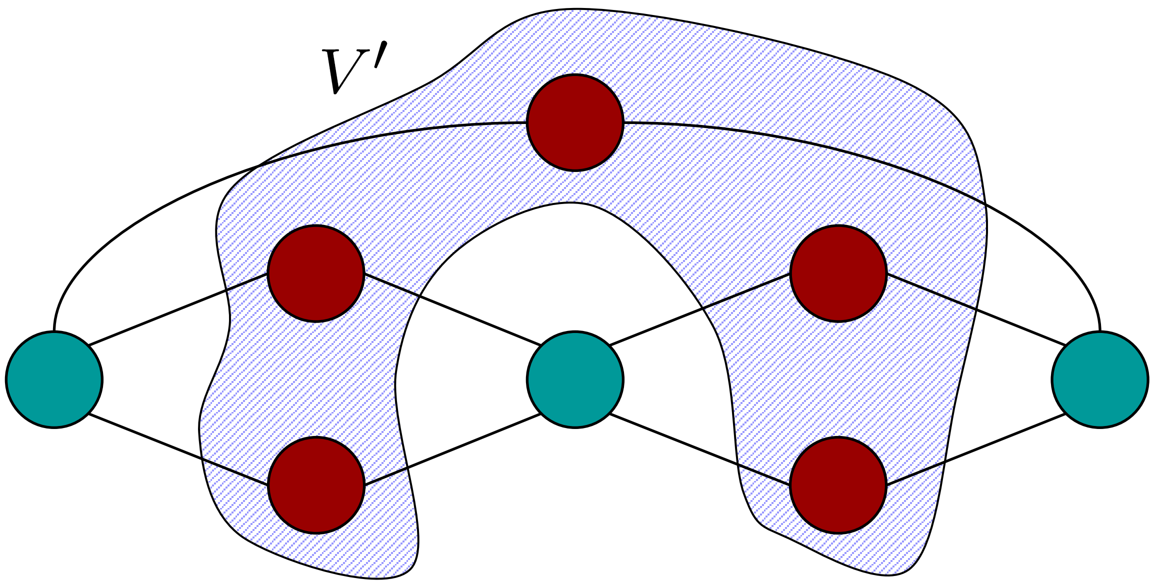
A set of *colour-patterns* is a *colouring-scheme*.

Model-Word Class

A subset $V' \subseteq V$ of $M = \langle V, E, \sigma \rangle$ that makes $M, w \models \delta$ for all $\delta \in \Delta$ and hence $M, w \models \varphi$, viz $\forall \delta \in \Delta$. $M, w \models \delta$ iff $w \in V'$.

The pair $\langle M, V' \rangle$ constitutes *Class*($\Gamma \models \Delta \Rightarrow \varphi$)

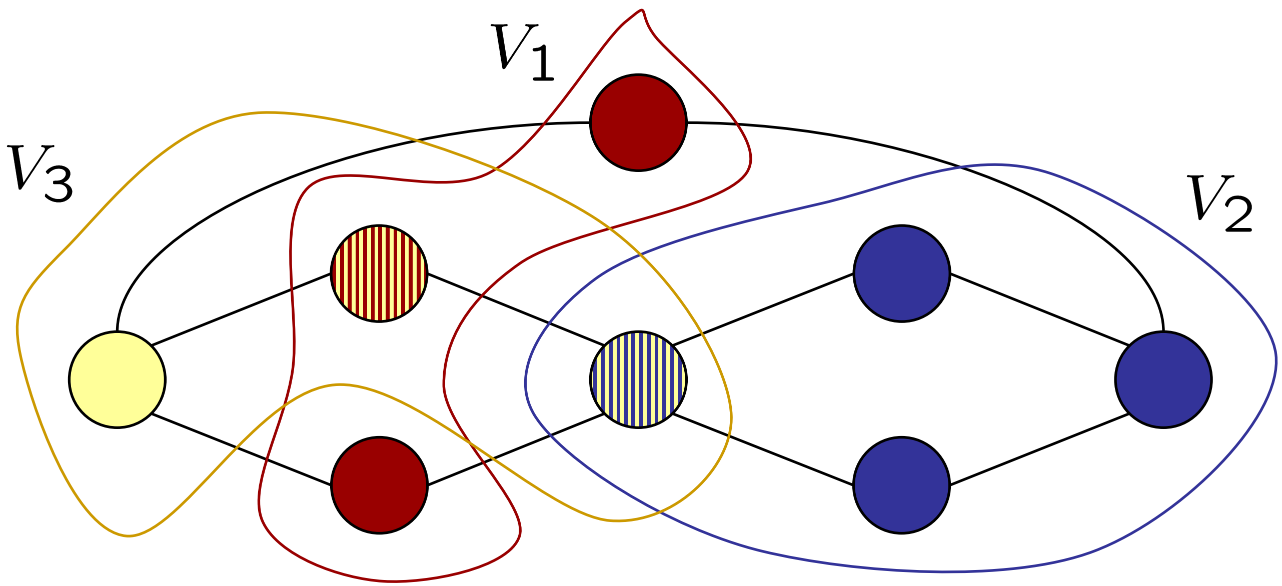
$\langle \langle V, E, \sigma \rangle, V' \rangle$ *Class*($Green \vee Red \models Green \Rightarrow \Box(Red)$)



Concretising Colouring-Schemas

A model $M = \langle V, E, \sigma \rangle$ *concretises* a colouring-scheme $\mathsf{S} = \{\Gamma_1 \models \Delta_1 \Rightarrow \varphi_1, \ldots, \Gamma_n \models \Delta_n \Rightarrow \varphi_n\}$, denoted by $M :: \mathsf{S}$, iff:

$$\forall(\Gamma_i \models \Delta_i \Rightarrow \varphi_i) \in \mathsf{S}.\ \exists \emptyset \subseteq V_i \subseteq V.$$
$$\langle M, V_i \rangle \in \mathit{Class}(\Gamma_i \models \Delta_i \Rightarrow \varphi_i) \text{ and } \bigcup V_i = V$$

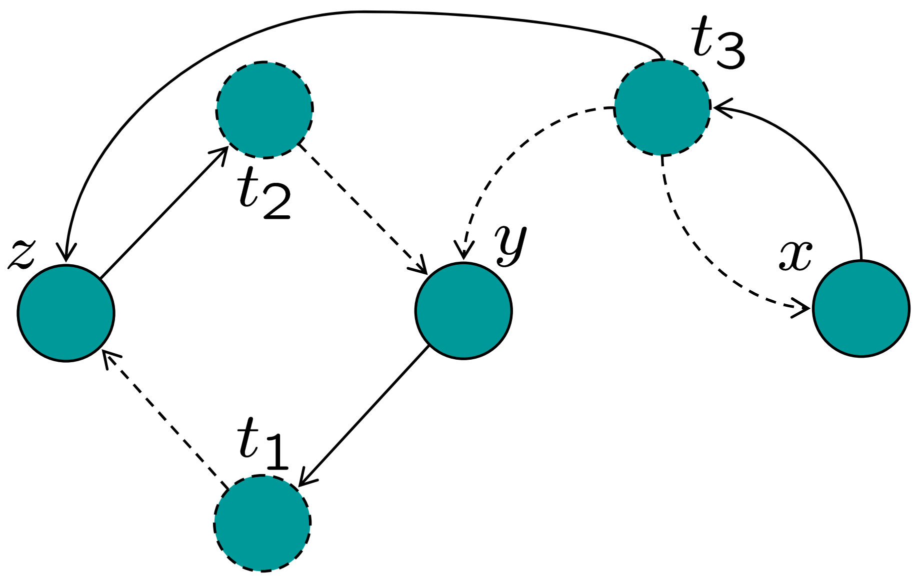


Classical Colouring

$$\mathsf{C} = \{\ \Gamma \models k_i \wedge k_0 \Rightarrow \Box^{\iota}(\neg k_2),$$
$$\Gamma \models k_1 \Rightarrow \top,$$
$$\Gamma \models k_t \wedge k_2 \Leftrightarrow \Box^{\iota} k_1 \wedge \Box^{\tau} k_0\ \}$$

$$\Gamma = ((k_i \wedge (k_1 \vee k_0)) \vee (k_t \wedge (k_2 \vee k_1))) \wedge \neg(k_i \wedge k_t)$$

$M :: \mathsf{C}$



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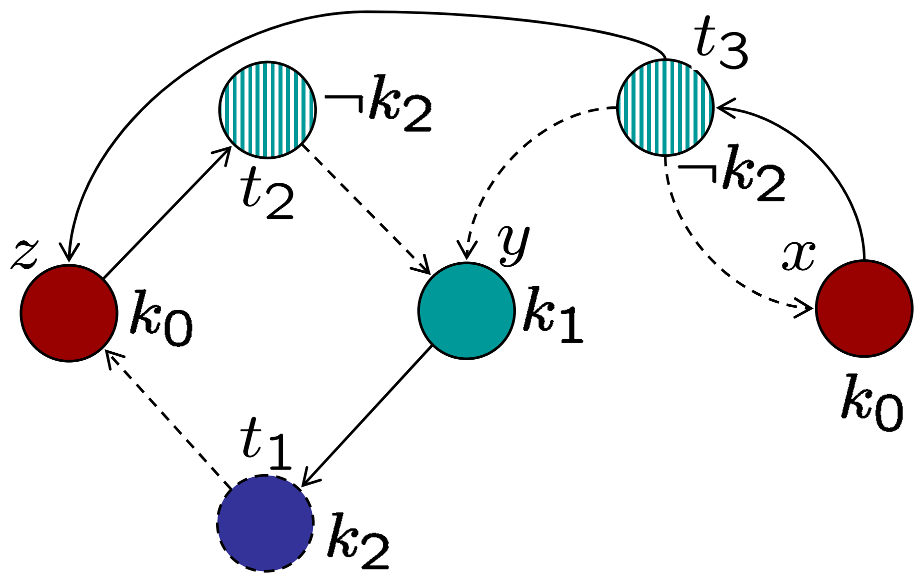

Pnueli & Shalev Statecharts

$\Phi := t_1 \wedge t_2 \wedge t_3$

$t_1 := \neg z \supset y$ ←

$t_2 := \neg y \supset z$ ←

$t_3 := (z \wedge \neg y \wedge \neg x) \supset x$

$\emptyset$

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$$\emptyset \longrightarrow \{z\}$$

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$$\emptyset \longrightarrow \{z\} \longrightarrow \{z, x\}$$

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$$t_3 := (z \wedge \neg y \wedge \neg x) \supset x \quad \longleftarrow \quad \times$$

$$\emptyset \longrightarrow \{z\} \longrightarrow \{z, x\}$$ **Backtrack**

Pnueli & Shalev Statecharts

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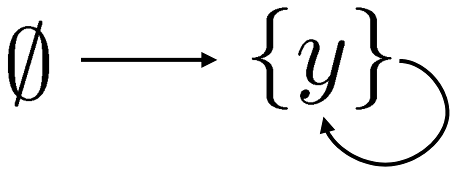

Intuitionistic Colouring

$$\mathsf{I} = \{\ \Gamma' \models k_3 \Rightarrow k_1,$$
$$\Gamma' \models k_1 \wedge \Diamond^{\iota} k_4 \Rightarrow k_3,$$
$$\Gamma' \models k_4 \Leftrightarrow k_2 \wedge \Box^{\iota} k_3,$$
$$\Gamma' \models k_5 \Leftrightarrow \neg(k_3 \vee k_4)\ \}$$

$$M = \langle V, E, \sigma \rangle$$
$$M' = \langle V, E, \sigma \cup \sigma' \rangle$$

$$M :: \mathsf{C} \wedge M' :: \mathsf{I}$$

$$\Gamma' = ((k_i \wedge (k_3 \vee k_5)) \vee (k_t \wedge (k_4 \vee k_5))$$

$$\sigma_i(k_1) = \sigma_j(k_1)$$

$$M_i \leq M_j \longrightarrow M :: \mathsf{C} \wedge (M' :: \mathsf{I})_{min}$$

$$\sigma'_i(k_3) \subseteq \sigma'_j(k_3)$$

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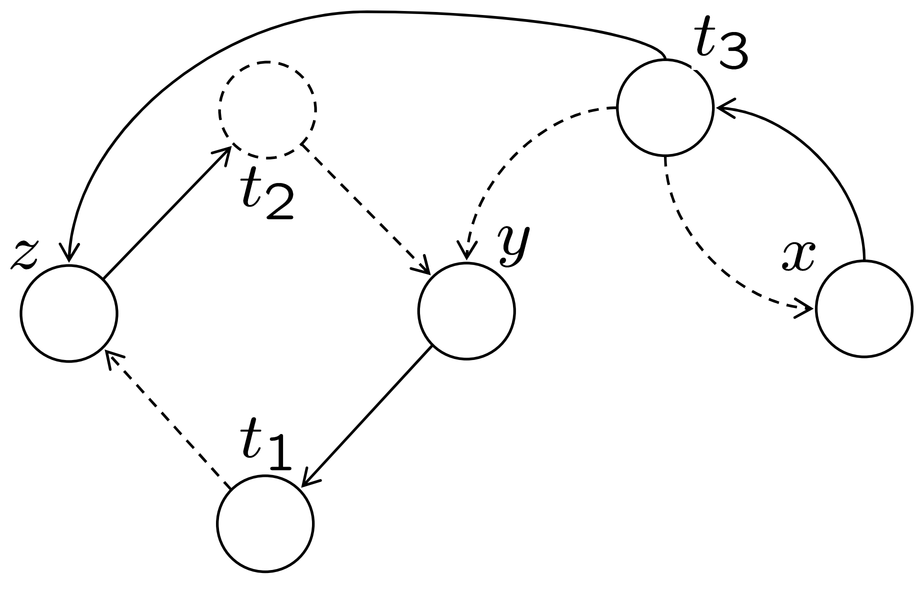

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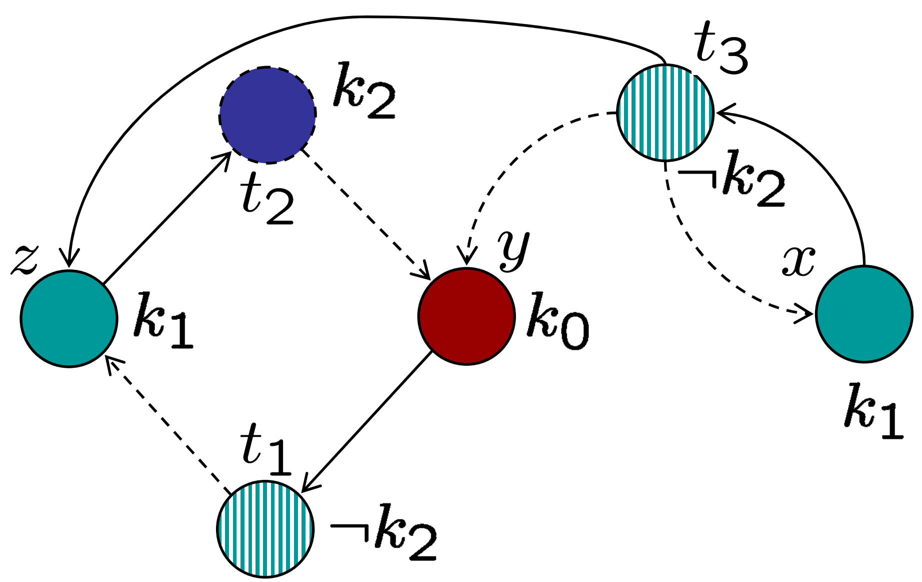

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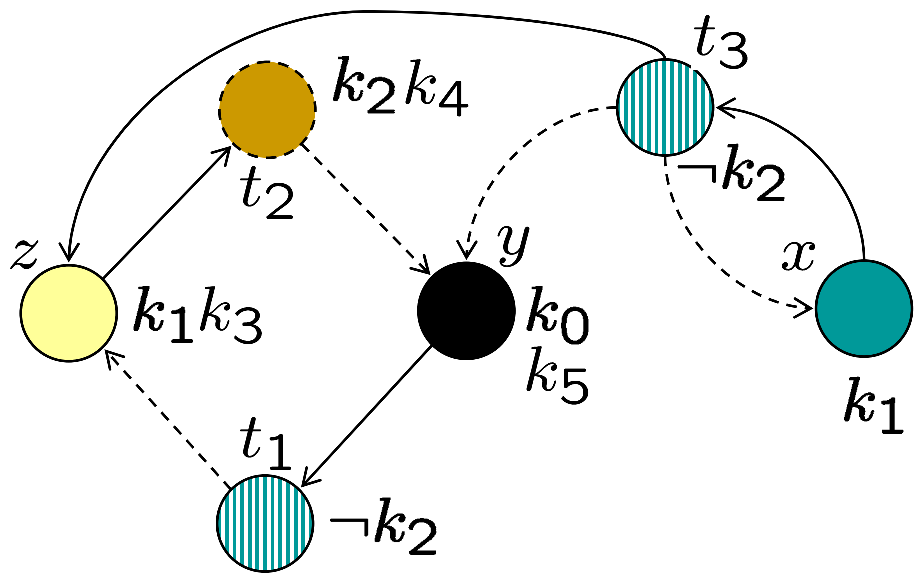

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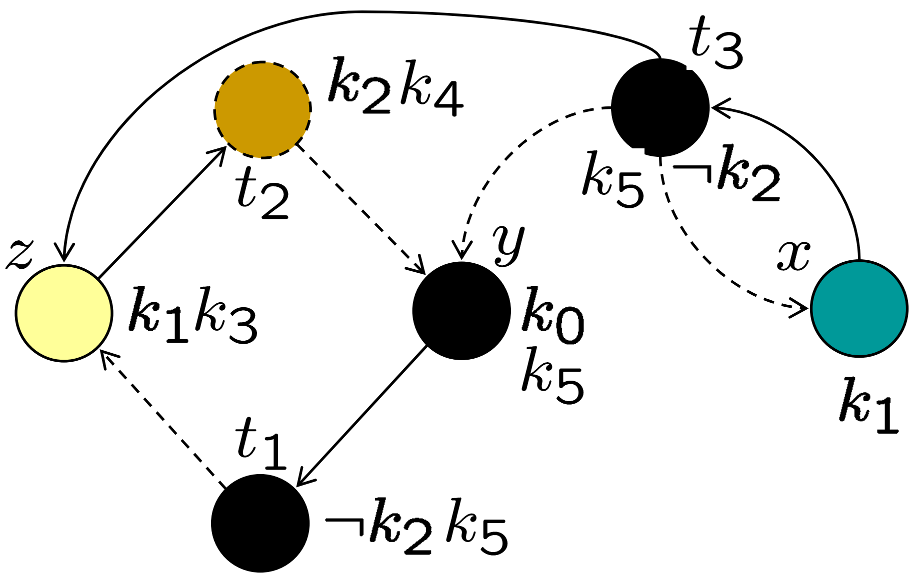

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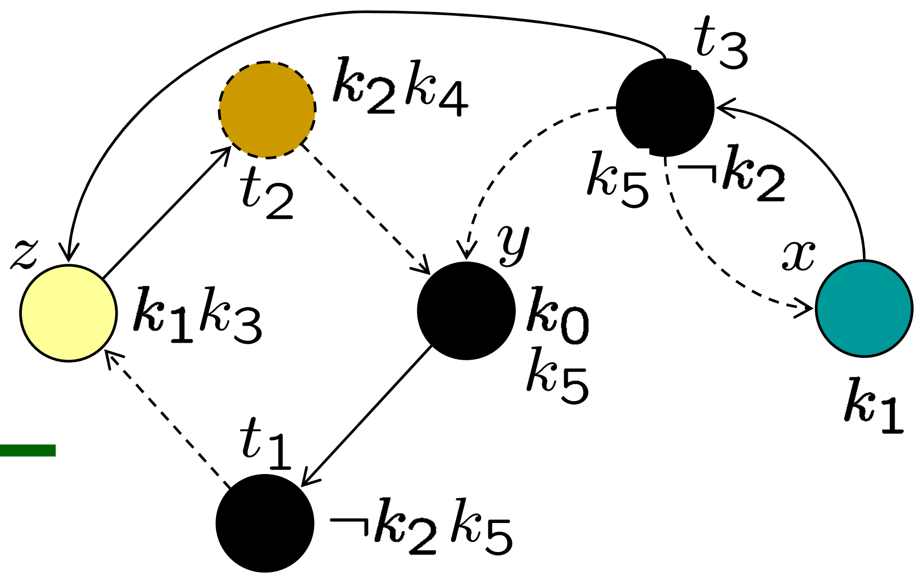

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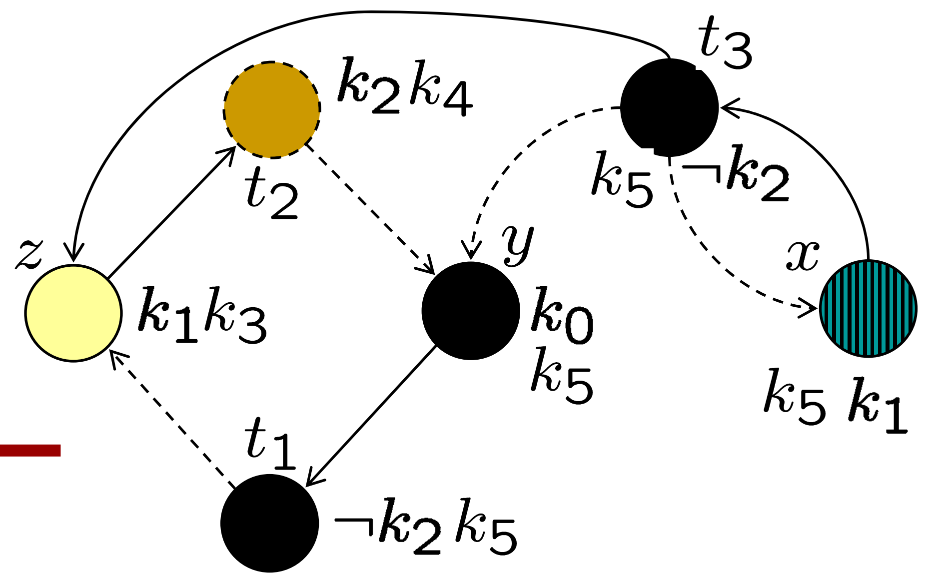

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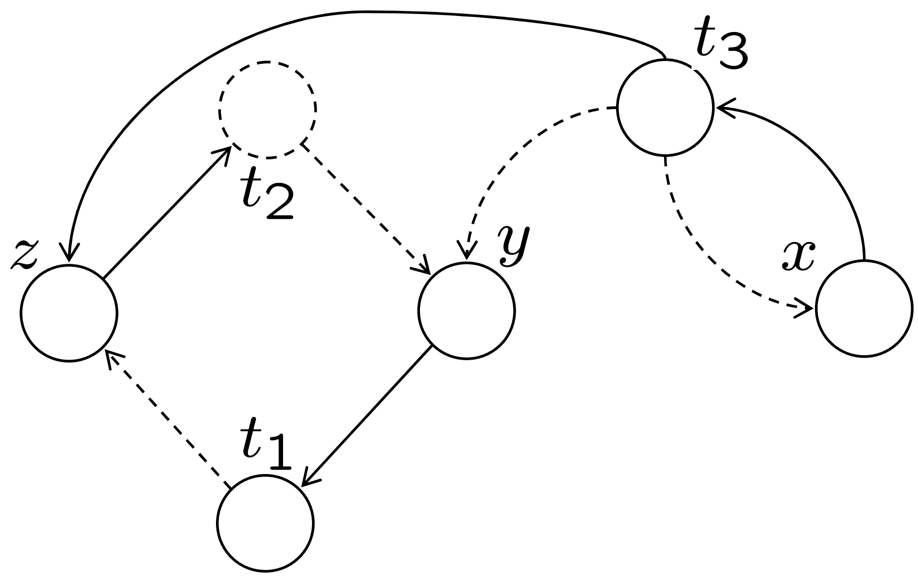

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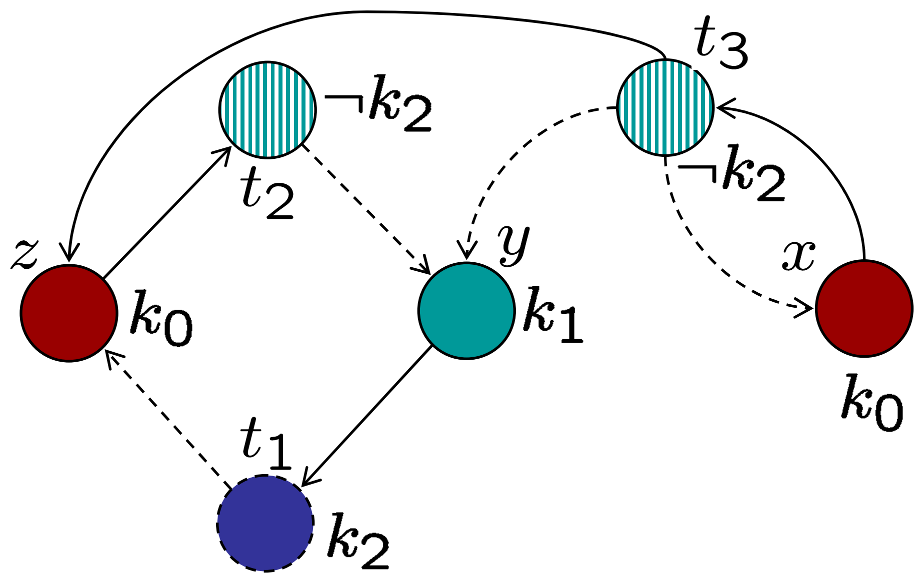

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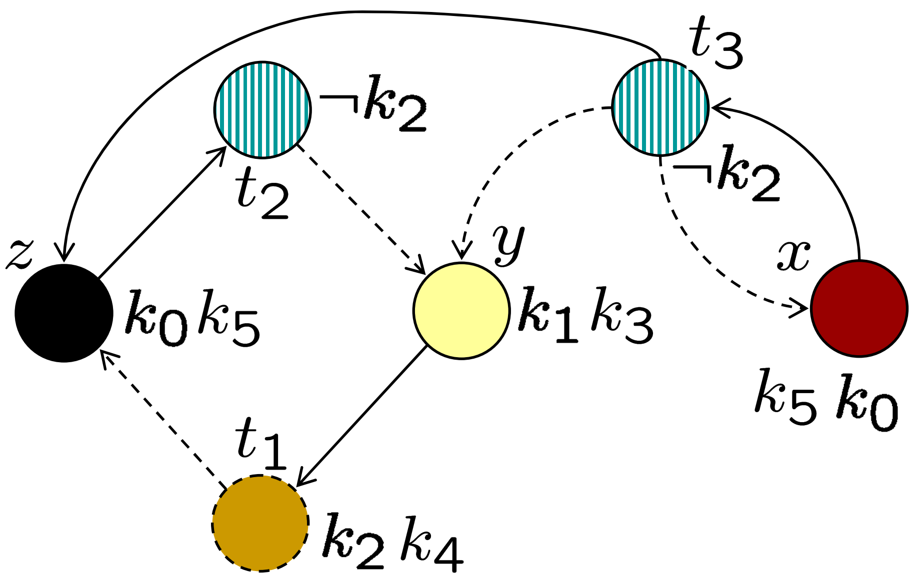

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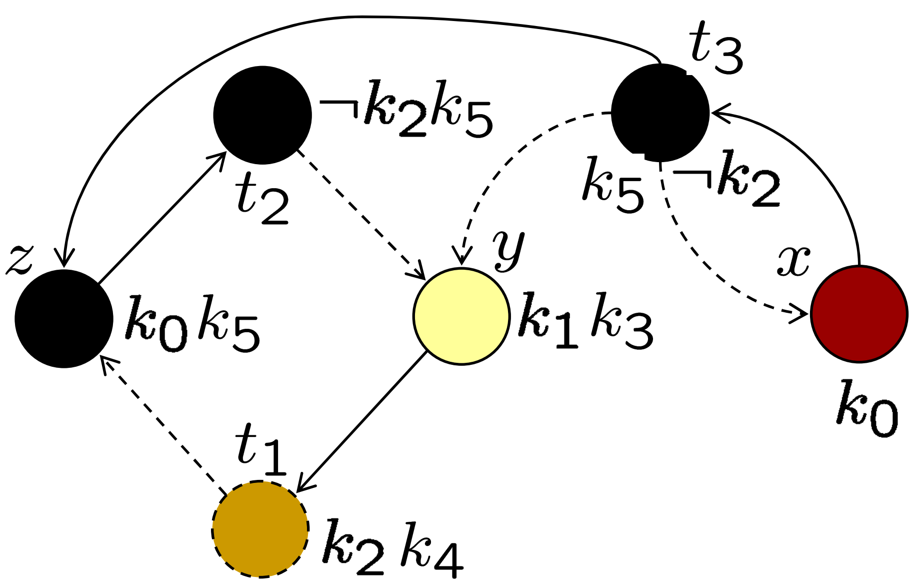

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$$\mathsf{P\&S} = \{\ \models k_3 \Leftrightarrow k_1,$$
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$$M :: \mathsf{C} \wedge (M' :: \mathsf{I})_{min} \wedge M' :: \mathsf{P\&S}$$

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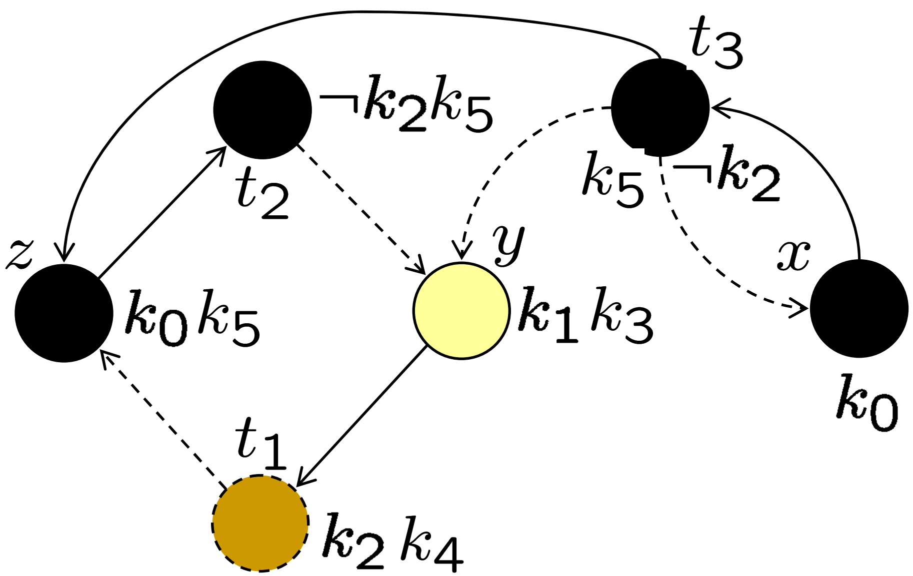

Summary

This Colouring semantics corresponds to the macro-step semantics of Pnueli & Shalev.

Graph colouring has concentrated on the optimisation (in the sense of minimality) under the proper colouring condition (i.e., there are no neighbours with the same colour) that despite all the efforts remains a very difficult combinatorial problem.

Summary

What our semantics is an immediate domain of application for graph colouring algorithms with weaker colouring conditions.

Practical application will include, for example, semantic-based program transformations.

Summary

On the other hand, every algorithm implementing the synchronous semantics discussed here is automatically one colouring algorithm complying specific constraints.

Also, not having considered other synchronous language we must leave open questions regarding the possibility of extend our framework in particular to ESTEREL.